Lattice calculation of BSM Kaon Bag parameters using improved staggered quark in $N_f = 2 + 1$ QCD.

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Phenomenology of $K^0 - \bar{K}^0$ mixing and CP violation

- Neutral Kaon System with two flavour eigenstates

$$|K^0\rangle, \qquad CP|K^0\rangle = |\bar{K}^0\rangle \tag{1}$$

- Weak-interaction-induced transition in the $K^0 - \bar{K}^0$ system can be described by S-matrix

$$S_{ab} = \langle a|T\mathrm{exp}(-i\int H_w'(t)dt)|b\rangle \tag{2}$$

where, $a, b = K^0$ or $\bar{K}^0$ and $H_w' = e^{iHt}H_w e^{-iHt}$.

- Neutral Kaon eigenstates K_S and K_L has mixing of CP eigenstates K_+(even) and K_-(odd).

$$K_S = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_+ + \bar{\varepsilon}K_-) \qquad K_L = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_- + \bar{\varepsilon}K_+)$$

Indirect CP violation and ϵ_K

- CP violation $K_L \to \pi\pi$ occurs in two ways
  1. $\bar{\varepsilon} K_+$(CP even) $\to \pi\pi$(CP even) : Indirect CP violation

$$\varepsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} \tag{3}$$

  2. K_-(CP odd) $\to \pi\pi$(CP even) : Direct CP violation

$$\varepsilon' = \frac{1}{\sqrt{2}}\varepsilon_K \left(\frac{\mathcal{A}[K_L \to (\pi\pi)_{I=2}]}{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]} - \frac{\mathcal{A}[K_S \to (\pi\pi)_{I=2}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} \right) \tag{4}$$

- Experimental Value of indirect CP violation parameter ε_K

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}, \quad \phi_\varepsilon = 43.52(5)^\circ. \tag{5}$$

$K^0 - \bar{K}^0$ Mixing in the Standard model

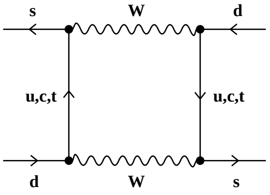
- Standard Model

$$\mathcal{H}_{eff}^{\Delta S=2} = C_1(\mu) Q_1(\mu) \qquad \mu \ll M_W \tag{6}$$

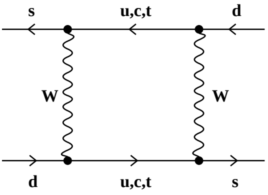
where four fermion operator Q_1 is given by

$$Q_1 = 4[\bar{s}\gamma_\mu P_L d][\bar{s}\gamma_\mu P_L d] \tag{7}$$

and $C_1 \propto G_F^2 M_W^2$



(a)



(b)

BSM Four Fermion Operators

- New $\Delta S = 2$ four-fermion operators that contribute to Kaon Mixing

$$
\begin{aligned}
Q_1 &= [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] \quad \rightarrow \quad B_K \\
Q_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] \\
Q_3 &= [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b] \\
Q_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
Q_5 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]
\end{aligned}
$$

- $\mathcal{H}_{eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu)$
- With the constraint from experiment, calculating corresponding hadronic matrix elements

$$\langle \bar{K}_0 | Q_i | K_0 \rangle$$

can impose strong constraints on BSM models.

Lattice Calculation : BSM B-parameters

- **B-parameters**

$$B_K = \frac{\langle \bar{K}_0 | Q_1 | K_0 \rangle}{8/3 \langle \bar{K}_0 | \bar{s}\gamma_0\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_0\gamma_5 d | K_0 \rangle} \qquad \text{SM, BSM}$$

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle} \qquad \text{BSM}$$

Where, $i = 2, 3, 4, 5$ and $(N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3)$

- **Golden Combinations :** G_i

$$G_{23} \equiv \frac{B_2}{B_3} \qquad G_{45} \equiv \frac{B_4}{B_5}$$

$$G_{24} \equiv B_2 \cdot B_4 \qquad G_{21} \equiv \frac{B_2}{B_K}$$

1. Advantage: no SU(2) chiral logs at NLO order in G_i (Golden Combinations)

Lattice QCD

- Failure of perturbation theory in low energy QCD. (For hadronic processes at scales ≤ 1GeV, $\alpha_s \simeq 1$.)
- Lattice QCD : In 1974 Wilson formulated Euclidean gauge theories on the space-time lattice.
  1. Discretize space-time.
  2. Define gauge-invariant lattice action.
  3. Construct the lattice operators.
  4. Define the path integral.
- Euclidean correlators on lattice.

$$\langle \mathcal{O}_2(t)\mathcal{O}_1(0)\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi}]\mathcal{D}[U]e^{-S_F-S_G}\,\mathcal{O}_2(t)\mathcal{O}_1(0) \qquad (8)$$

Staggered Quark

- Fermion doubling problem : In d-dimension, naive Wilson fermion has $2^d - 1$ unwanted-pole(doubler) in the fermion propagater.

$$S^{-1}(p) = m_q + \frac{i}{a} \sum_{\mu} \gamma_\mu \mathsf{sin} p_\mu a \tag{9}$$

- Nielsen-Ninomiya theorem : We can not make doubler-free lattice action with chiral symmetry.

$$D\gamma_5 + \gamma_5 D = 0 \tag{10}$$

- Staggered Quark : Reduce doublers by 4 using spin-diagonalization.

$$\psi(x) = \Gamma_x \chi(x) \quad \bar{\psi}(x) = \bar{\chi}(x)\Gamma_x^\dagger \quad \Gamma_x = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}. \tag{11}$$

where, $x_1, \cdots, x_4$ is the lattice site.

$N_f = 2 + 1$ QCD: MILC fine lattices - Staggered Quarks

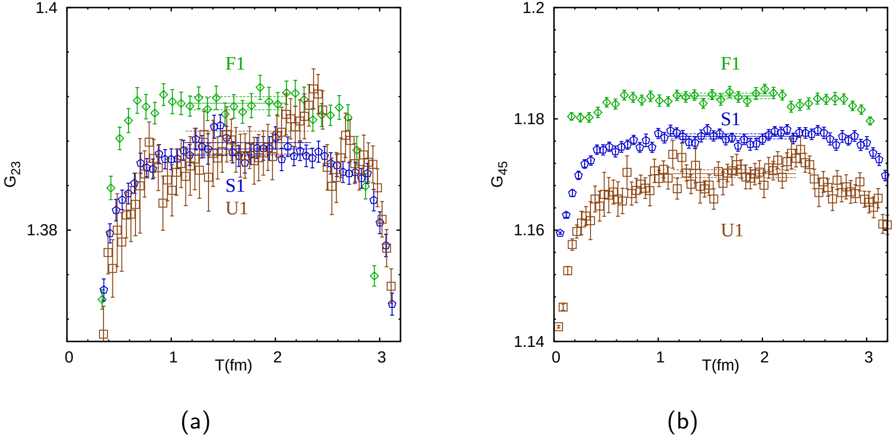
a (fm)	am_l/am_s	geometry	ens×meas	ID	Status
0.09	0.0062/0.0310	$28^3 \times 96$	995×9	F1	
0.09	0.0031/0.0310	$40^3 \times 96$	959×9	F2	
0.09	0.0093/0.0310	$28^3 \times 96$	949×9	F3	
0.09	0.0124/0.0310	$28^3 \times 96$	1995×9	F4	
0.09	0.00465/0.0310	$32^3 \times 96$	651×9	F5	
0.09	0.0062/0.0186	$28^3 \times 96$	950×9	F6	New
0.09	0.0031/0.0186	$40^3 \times 96$	701×9	F7	New
0.09	0.00155/0.0310	$64^3 \times 96$	790×9	F9	New
0.06	0.0036/0.018	$48^3 \times 144$	749×9	S1	
0.06	0.0025/0.018	$56^3 \times 144$	799×9	S2	
0.06	0.0072/0.018	$48^3 \times 144$	593×9	S3	
0.06	0.0054/0.018	$48^3 \times 144$	582×9	S4	
0.06	0.0018/0.018	$64^3 \times 144$	572×9	S5	New
0.045	0.0030/0.015	$64^3 \times 192$	747×1	U1	

Data Analysis

- **Calculate raw data**
  Calculate B_K and G_i for different valence quark mass combinations for each gauge ensemble. ($\overline{\text{MS}}$ scheme with NDR.)
- **Chiral fitting**
  X-fit: Fix valence strange quark mass and extrapolate the light quark mass m_x to physical down quark mass.
  Y-fit: Extrapolate m_y to physical strange quark mass.
- **RG Evolution**
  Obtain results at μ_f =2GeV or 3GeV by running from $\mu_i = 1/a$.
- **Continuum extrapolation**
  Perform [1–3] for different lattices and extrapolate to $a = 0$ and to physical sea quark masses.

Raw Data of G_{23} and G_{45}

- We compare three ensembles which have the same ratio of sea quark mass $m_\ell/m_s = 1/5$.



SchPT X-fit and Y-fit of G-parameter

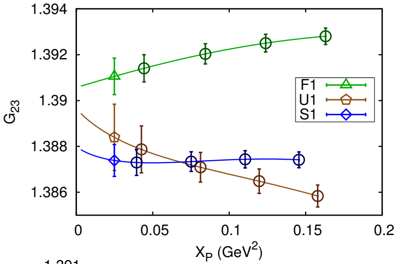
- NNNLO X-fit

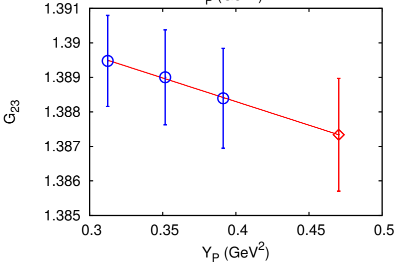
$$
\begin{aligned}
G_i & \quad (\text{NNNLO}) \\
= & \quad c_1 + c_2 X + c_3 X^2 \\
+ & \quad c_4 X^2 (\ln(X))^2 \\
+ & \quad c_5 X^2 \ln(X) + c_6 X^3
\end{aligned}
$$

Bayesian constraints on $c_{4-6} = 0 \pm 1$.

- Y-fit(U1 ensemble)

$$G_i(\text{Y-fit}) = b_1 + b_2 Y_P$$





Chiral-Continuum Fit

- We use 14 data points from 14 MILC ensembles in the fitting. We extrapolate the results to physical point $a \to 0$, $L_P \to L_P^{\text{phy}} = m_{\pi_0}^2$, and $S_P \to S_P^{\text{phy}} = m_{s\bar{s}}^2$.
- Fitting functional forms come from the SU(2) SChPT theory.

fit type	fitting functional form	Bayesian Constraints
$\tilde{F}_B^1$	$d_1 + d_2 \frac{L_P}{\Lambda_\chi^2} + d_3[\frac{S_P - S_P^{\text{phy}}}{\Lambda_\chi^2}] + d_4(a\Lambda_Q)^2$	$d_2, d_3, d_4 = 0 \pm 2$
$\tilde{F}_B^4$	$\tilde{F}_B^1 + d_5(a\Lambda_Q)^2 \frac{L_P}{\Lambda_\chi^2} + d_6(a\Lambda_Q)^2[\frac{S_P - S_P^{\text{phy}}}{\Lambda_\chi^2}]$ $+ d_7(a\Lambda_Q)^2\alpha_s + d_8\alpha_s^2 + d_9(a\Lambda_Q)^4$	$d_2 \cdots d_9 = 0 \pm 2$
$\tilde{F}_B^6$	$\tilde{F}_B^4 + d_{10}\alpha_s^3 + d_{11}(a\Lambda_Q)^2\alpha_s^2 + d_{12}(a\Lambda_Q)^4\alpha_s$ $+ d_{13}(a\Lambda_Q)^6 + d_{14}(a\Lambda_Q)^4 \frac{L_P}{\Lambda_\chi^2} + d_{15}(a\Lambda_Q)^2\alpha_s \frac{L_P}{\Lambda_\chi^2}$ $+ d_{16}\alpha_s^2 \frac{L_P}{\Lambda_\chi^2} + d_{17}(a\Lambda_Q)^4[\frac{S_P - S_P^{\text{phy}}}{\Lambda_\chi^2}]$ $+ d_{18}(a\Lambda_Q)^2\alpha_s[\frac{S_P - S_P^{\text{phy}}}{\Lambda_\chi^2}] + d_{19}\alpha_s^2[\frac{S_P - S_P^{\text{phy}}}{\Lambda_\chi^2}]$	$d_2 \cdots d_{19} = 0 \pm 2$

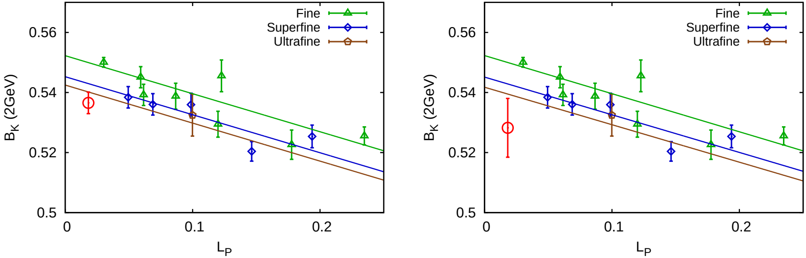
Chiral-Continuum Fit : Fitting quality

- We can see that the χ^2 values for fitting functional forms get saturated as we add higher order terms in the fitting functional forms. We choose $\tilde{F}_B^1$-fit results as central values for B_K, G_{24}, and G_{21}. For G_{23} and G_{45}, we choose those of $\tilde{F}_B^4$ as the central values.

fit type	B_K	G_{21}	G_{23}	G_{24}	G_{45}
$\tilde{F}_B^1$	1.49	1.25	2.01	1.08	4.07
$\tilde{F}_B^4$	1.48	1.18	1.33	0.91	1.39
$\tilde{F}_B^6$	1.48	1.13	1.26	0.89	1.29

Chiral-Continuum Fit of B_K

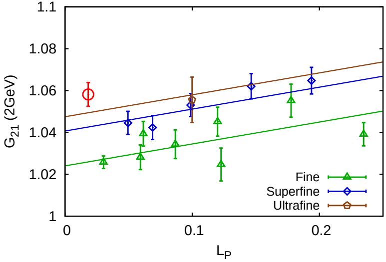
- The result of Chiral-Continuum fit. The straight line in the plots represents the value of fitting function at fixed S_P and a^2 for fine($a \approx 0.09$fm), superfine ($a \approx 0.06$fm), and ultrafine($a \approx 0.045$fm) gauge ensembles.



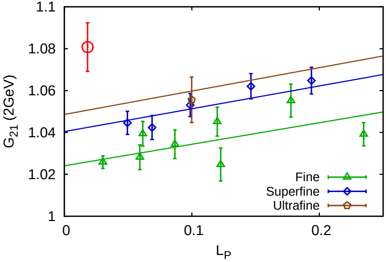
(c) $\tilde{F}_B^1$, $\chi^2/\text{dof} = 1.49$

(d) $\tilde{F}_B^4$, $\chi^2/\text{dof} = 1.48$

Chiral-Continuum Fit of G_{21}

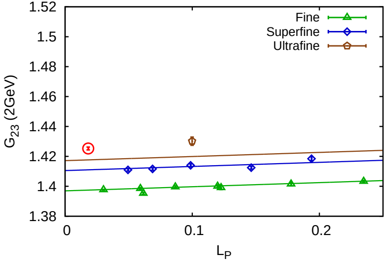


(e) $\tilde{F}_B^1$, $\chi^2/\text{dof} = 1.25$

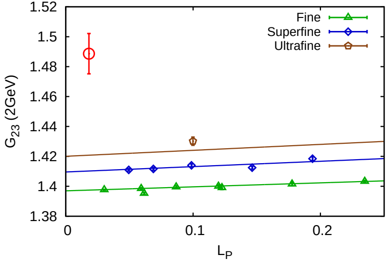


(f) $\tilde{F}_B^4$, $\chi^2/\text{dof} = 1.18$

Chiral-Continuum Fit of G_{23}

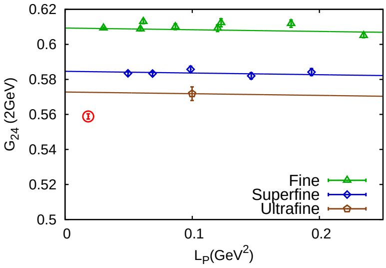


(g) $\tilde{F}_B^1$, χ^2/dof $= 2.01$

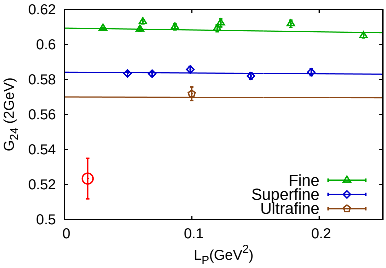


(h) $\tilde{F}_B^4$, χ^2/dof $= 1.33$

Chiral-Continuum Fit of G_{24}

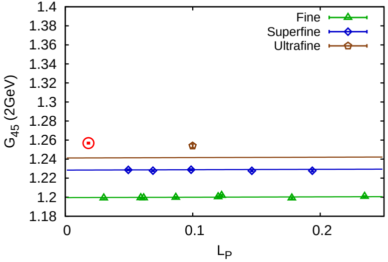


(i) $\tilde{F}_B^1$, χ^2/dof $= 1.08$

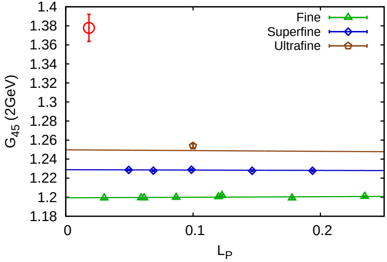


(j) $\tilde{F}_B^4$, χ^2/dof $= 0.91$

Chiral-Continuum Fit of G_{45}



(k) $\tilde{F}_B^1$, χ^2/dof $= 4.07$



(l) $\tilde{F}_B^4$, χ^2/dof $= 1.39$

Results and Comparison

- We obtain B_i from results of G_i and B_K. (Indicated errors are preliminary.)
- Comparing results to those of other collaborations, there are discrepancies in $B_2(\sim 2\sigma)$, B_4 and $B_5(\sim 3\sigma)$.
  1. SWME : $N_f = 2+1$ staggered quark, one-loop matching.[Bae et al.(2013)]
  2. RBC & UKQCD : $N_f = 2+1$ domain-wall quark, non-perturbative matching.[Boyle et al.(2012)Boyle, Garron, and Hudspith]
  3. ETM : $N_f = 2$ maximally twisted sea quark and Osterwalder-Seiler valence quarks, non-perturbative matching.[Bertone et al.(2013)]

	SWME		RBC&UKQCD	ETM
	$\mu = 2$GeV	$\mu = 3$GeV	$\mu = 3$ GeV	$\mu = 3$ GeV
B_K	0.54(3)	0.52(3)	0.53(2)	0.51(2)
B_2	0.57(3)	0.52(2)	0.43(5)	0.47(2)
B_3^{Buras}	0.38(2)	0.36(2)	N.A.	N.A.
B_3^{SUSY}	0.85(5)	0.77(5)	0.75(9)	0.78(4)
B_4	0.98(6)	0.98(6)	0.69(7)	0.75(3)
B_5	0.71(7)	0.75(7)	0.47(6)	0.60(3)

Error-Budget

cause	B_K	B_2	B_3	B_4	B_5
statistics	0.67	0.25	0.95	0.27	1.08
chiral-cont extrap and matching	4.40	4.40	4.45	6.46	9.63
X-fits	0.05	0.36	0.35	0.37	1.23
Y-fits	2.07	0.04	0.36	0.45	1.56
finite volume	0.74	0.70	0.05	0.43	0.03
f_π	0.10	0.10	0.10	0.10	0.10
Total	**4.97**	**4.48**	**4.58**	**6.51**	**9.89**

Table: Error budget for B-parameters B_i evaluated at $\mu = 2$GeV (unit : %) using SU(2) SChPT fitting.

Summary

- The total errors of BSM B-parameters are $5-10\%$ level, which are mainly from the chiral-continuum extrapolation and the matching.
- The B_K parameter agrees with the results from RBC & UKQCD and ETM collaboration.
- In the case of BSM B-parameter, there is $2\sigma(B_2)$ to $3\sigma(B_4$ and $B_5)$ discrepancy beween our result and those from RBC & UKQCD and ETM collaboration.
- We guess that the discrepancy comes from the difference in matching. We use perturbative matching, whereas RBC & UKQCD and ETM collaboration use NPR (non-perturbative renormalization).
- To confirm our guess, we will obtain the matching factor using NPR (Jangho Kim) in the near future.

Bibliography

- T. Bae et al. (SWME Collaboration), Phys.Rev. **D88**, 071503 (2013), 1309.2040.
- P. Boyle, N. Garron, and R. Hudspith (RBC Collaboration, UKQCD Collaboration), Phys.Rev. **D86**, 054028 (2012), 1206.5737.
- V. Bertone et al. (Collaboration ETM), JHEP **1303**, 089 (2013), 1207.1287.